Spherical Linear Interpolation for Transmit Beamforming in MIMO-OFDM Systems with Limited Feedback

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Abstract Transmit beamforming with receive combining is a simple approach to exploiting the significant diversity provided by multiple-input multiple-output (MIMO) systems, and this technique can be easily extended to frequency selective MIMO channels by employing orthogonal frequency division multiplexing (OFDM). Optimal beamforming requires channel state information in the form of the beamforming vectors corresponding to all the OFDM subcarriers. When the uplink and downlink channels are not reciprocal, this information must be conveyed back to the transmitter.

To reduce the amount of feedback information, a new approach to transmit beamforming is proposed that combines limited feedback and beamformer interpolation. Because the length of the OFDM cyclic prefix is designed to be much less than the number of subcarriers to increase spectral efficiency, the neighboring subchannels of a MIMO-OFDM system are substantially correlated. Thus, the beamforming vectors determined by the subchannels are also significantly correlated. To reduce the feedback information using the correlation between beamforming vectors, the receiver of the proposed scheme sends back only a fraction of information about the optimal beamforming vectors to the transmitter. Then the transmitter evaluates the beamforming vectors for all subcarriers through interpolation of the conveyed beamforming vectors. Since a beamforming vector is phase invariant and has unit norm, a new spherical linear interpolator is proposed that exploits additional parameters for phase rotation. These parameters are determined at the receiver in the sense of maximizing the minimum channel gain or capacity, and they are sent back to the transmitter along with the beamforming vectors through the feedback channel.

For practical implementation, the proposed interpolator with quantized phase rotations is combined with existing approaches for beamforming vector quantization. The performance of the proposed beamforming scheme is found to outperform comparable beamforming techniques in terms of bit error rate (BER) and capacity and yet requires less channel state information.

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***Abstract*— Transmit beamforming and receive combining can be simply implemented over frequency selective channels by using multiple-input multiple-output (MIMO) technology combined with orthogonal frequency division multiplexing (OFDM) in what is known as MIMO-OFDM. Generally this requires knowledge of the transmit beamforming vectors for every tone at the transmitter. In systems without reciprocity between the uplink and the downlink channels, this requires informing the transmitter about all the beamforming vectors through a feedback control channel. To reduce the feedback required, this paper proposes to exploit channel coherence to send back only a fraction of the beamforming vectors. The coherence of the transmit beamforming vectors is established. A new spherical interpolator, with an additional parameter for phase rotation, is proposed to fill in the gaps between known vectors. The parameters of the interpolator are optimized in the sense of maximizing mutual information and are sent back to the transmitter along with the beamforming vectors through the feedback channel. Simulation results show that the proposed scheme improves performance and reduces feedback requirements compared to existing beamforming techniques.**

1. Introduction

The spatial dimension of multiple-input multiple-output (MIMO) systems can be used to mitigate signal-level fluctuations in fading channels through thoughtful exploitation of antenna diversity. In a MIMO system, diversity can be obtained by employing space-time codes [1]–[3] without transmit channel knowledge or alternatively by using the channel state information (CSI) at the transmitter using transmit beamforming and receive combining [4]–[8]. Compared with space-time coding, beamforming and combining achieve full diversity order, they work with any number of transmit and receive antennas, and they achieve additional array gain.

Diversity techniques proposed for narrowband channels can be easily extended to frequency selective MIMO channels by employing orthogonal frequency division multiplexing (OFDM). In a MIMO-OFDM system with channel state information [9], transmit beamforming with receive combining is performed independently for each subcarrier [9], [10]. This approach, however, requires the knowledge about the transmit beamforming vectors at the transmitter. In non-reciprocal channels, this necessitates that the receiver informs the transmitter about the beamforming vectors for all subcarriers through a feedback control channel. A practical solution for reducing the amount of feedback per beamforming vector is to use a codebook designed for quantization of the transmit beamforming vectors (see e.g., [11]–[13]). The feedback requirements, though, still increase in proportion to the number of subcarriers.

In an attempt to reduce the feedback requirements, in this paper we propose a new approach to transmit beamforming that combines limited feedback of beamforming information and interpolation of beamforming vectors. In OFDM, the number of subcarriers is designed to be much larger than the length of the discrete time domain channel impulse response to increase spectral efficiency (see e.g., [14], [15]). This causes significant correlation between neighboring OFDM subchannels which results in high correlation between beamforming vectors. We analyze the correlation between beamforming vectors and use this result to reduce the amount of feedback information. We propose to subsample the space of beamforming vectors and send back only the beamforming vectors for select subcarriers to the transmitter. The transmitter in turn determines the missing beamforming vectors through a smart interpolation of the conveyed beamforming vectors.

In this paper, we assume that the transmit power is identically assigned to all subcarriers. The beamforming vectors have unit norm by the transmit power constraint, thus the outputs of the interpolation problem should maintain the unit norm constraint. This is known as spherical interpolation [16]–[18]. Unfortunately, it is not easy to directly use spherical interpolation algorithms to interpolate beamforming vectors because the optimal beamforming vectors are not unique. As an alternative, we propose a new spherical linear interpolator with additional parameters for phase rotation, which consider the nonuniqueness of the optimal beamforming vector. Our interpolator can be used to reduce the feedback requirements for transmit beamforming and can be used to reduce complexity at the receiver by permitting the singular value decomposition to be calculated for only a fraction of tones. To maximize

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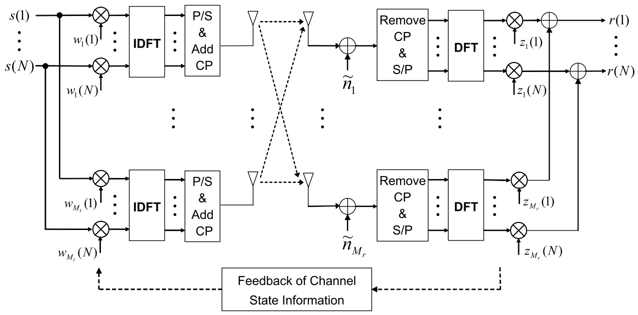

Fig. 1. Block diagram of a MIMO-OFDM system with M_t transmit antennas, M_r receive antennas, and N subcarriers.

the performance of the proposed interpolator, the phase parameters are determined in the sense of maximizing the capacity. In fact, it is not trivial to find the optimal solution from the cost function maximizing the capacity. Fortunately, we can use a numerical grid search or obtain a closed form solution through an approximation of the cost function. Computer simulations show that the proposed scheme outperforms existing diversity techniques with comparable feedback information and performs close to the ideal beamforming with feedback of all beamforming vectors.

For practical implementation, the proposed interpolation method can be combined with beamformer quantization techniques illustrated in [11]–[13]. In this case, the quantized beamforming vectors are used to find the optimal phases and perform the interpolation. We defer the details of quantizing the beamforming vector and the phase information are discussed in [19].

This paper is organized as follows. Section 2 introduces the MIMO-OFDM system with beamforming and combining. We presents the proposed interpolation based beamforming scheme and the phase optimization for the proposed spherical interpolator in Section 3. The proposed beamforming method is compared with existing diversity techniques in terms of bit error rate (BER) and capacity in Section 4. Finally, we provide some conclusions in Section 5.

2. System Overview

Fig. 1 illustrates a MIMO-OFDM system with transmit beamforming and receive combining with M_t transmit antennas, M_r receive antennas, and N subcarriers. During one OFDM symbol period, the scalar symbol $s(k)$ for the k-th subcarrier is mapped into the transmit antennas using the beamforming vector $\mathbf{w}(k) = [w_1(k), w_2(k), \cdots, w_{M_t}(k)]^T$. Assuming the sampled impulse response of the channel is shorter than the cyclic prefix (CP), the channel for the k-th subcarrier can be described by a M_r-by-M_t channel matrix $\mathbf{H}(k)$ whose entries represent the channel gains experienced by subcarrier k. At the subcarrier k, the received signal after processing with the combining vector $\mathbf{z}(k) = [z_1(k), z_2(k), \cdots, z_{M_r}(k)]^T$ can be expressed as

$$r(k) = \mathbf{z}^H(k)\{\mathbf{H}(k)\mathbf{w}(k)s(k) + \mathbf{n}(k)\}, \quad 1 \le k \le N \quad (1)$$

where $\mathbf{n}(k)$ is the M_r-dimensional noise vector whose entries have the independent and identically distributed (i.i.d.) complex Gaussian distribution with zero mean and variance N_0. We assume that the power is allocated equally across all subcarriers, thus $E[|s(k)|^2] = \mathcal{E}_s$ is a constant and $\|\mathbf{w}(k)\| = 1$ (where $\|(\cdot)\|$ means 2-norm of $(\cdot)$) to maintain the overall power constraint.

Since the signal model in (1) is identical to that of a narrowband MIMO system, $\mathbf{w}(k)$ and $\mathbf{z}(k)$ can be chosen to maximize the signal to noise ratio (SNR) using the results for narrowband MIMO systems [4], [6]. Without loss of generality, we can fix $\|\mathbf{z}(k)\| = 1$. Then the SNR for subcarrier k can be written as

$$\gamma(k) = \frac{\mathcal{E}_s |\mathbf{z}^H(k)\mathbf{H}(k)\mathbf{w}(k)|^2}{N_0} = \frac{\mathcal{E}_s}{N_0}\Gamma(k). \quad (2)$$

where $\Gamma(k) = |\mathbf{z}^H(k)\mathbf{H}(k)\mathbf{w}(k)|^2$ is the effective channel gain.

Given $\mathbf{w}(k)$, it is possible to show that the SNR maximizing solution uses maximum ratio combining (MRC) with

$$\mathbf{z}(k) = \frac{\mathbf{H}(k)\mathbf{w}(k)}{\|\mathbf{H}(k)\mathbf{w}(k)\|}. \quad (3)$$

On the other hand, given $\mathbf{z}(k)$, the effective channel gain $\Gamma(k)$ can be maximized through maximum ratio transmission (MRT). With MRT and MRC, $\mathbf{w}(k)$ is simply the dominant right singular vector of $\mathbf{H}(k)$ corresponding to the largest singular vector of $\mathbf{H}(k)$ [6]. The MRT/MRC scheme obtains full diversity order in Rayleigh fading channels and achieves the full array gain available. Unfortunately, this improved performance comes at the expense of knowledge of the beamforming vectors $\{\mathbf{w}(k), k = 1, 2, \cdots, N\}$. The extensive feedback required for the optimal MRT/MRC solution motivates developing new methods that achieve near maximum SNR performance but with more realistic feedback requirements.

3. Interpolation Based Beamforming for MIMO-OFDM

In this section, we analyze the correlation between subchannels of MIMO-OFDM and show that the correlation between the beamforming vectors is similar to the subchannel correlation in a multiple-input single-output (MISO) system. Next we use these results to motivate subsampling the beamforming vectors followed by beamformer interpolation. We introduce a beamformer interpolator based on spherical linear interpolator but with additional parameters for phase rotation, which takes the invariance of the optimal beamforming vectors into account. Finally, we optimize

the phase rotation parameters in the sense of maximizing mutual information.

3.1. Channel Correlation Between Subcarriers

Let us define the L-dimensional vector $\mathbf{g}_{i,j} = [g_{i,j}(0), g_{i,j}(2), \cdots, g_{i,j}(L-1)]^T$ which denotes the time-domain channel impulse response with length L between the transmit antenna j and the receiver antenna i. Suppose that $\{g_{i,j}(n)\}$ are i.i.d. complex Gaussian with zero mean and unit variance. For notational convenience, we define the $(M_t M_r)$-dimensional vector representing the channel gains for subcarrier k as

$$\mathbf{h}(k) = \mathrm{vec}(\mathbf{H}(k)) \tag{4}$$

where $\mathrm{vec}(\mathbf{X})$ means the vector obtained by stacking the columns of $\mathbf{X}$ on top of each other and $1 \le k \le N$. The channel vector $\mathbf{h}(k)$ is obtained by the discrete Fourier transform (DFT) of time-domain channel impulse responses and hence denoted by

$$\mathbf{h}(k) = \mathbf{G}^T \mathbf{f}_k \tag{5}$$

where $\mathbf{G} = [\mathbf{g}_{1,1}, \cdots, \mathbf{g}_{M_r,1}, \mathbf{g}_{1,2}, \cdots, \mathbf{g}_{M_r,2}, \cdots, \mathbf{g}_{M_r,M_t}]$ is the L-by-$M_t M_r$ matrix composed of all time-domain channel impulse responses between transmit and receive antenna pairs, and $\mathbf{f}_k$ is the first L elements of the k-th column of the DFT matrix which is given by

$$\mathbf{f}_k = \frac{1}{N}[1, e^{-j2\pi k/N}, e^{-j2\pi(2k)/N}, \cdots, e^{-j2\pi(L-1)k/N}]^T. \tag{6}$$

To measure the correlation between the subchannel k and the subchannel $(k+d)$, a new real variable $\rho_{\mathbf{h}}(d)$ is defined as

$$\rho_{\mathbf{h}}(d) = \frac{E[|\mathbf{h}^H(k+d)\mathbf{h}(k)|^2]}{E[\|\mathbf{h}(k)\|^4]} \tag{7}$$

where $0 \le \rho_{\mathbf{h}}(d) \le 1$. Suppose that the channel gains between different antenna pairs are independent, i.e. $E[\mathbf{G}^H\mathbf{G}] = L\mathbf{I}_L$. Then $\rho_{\mathbf{h}}(d)$ is expressed as

$$\begin{aligned} \rho_{\mathbf{h}}(d) &= \frac{\mathbf{f}_{k+d}^H \mathbf{P}(k) \mathbf{f}_{k+d}}{E[|\mathbf{h}^H(k)\mathbf{h}(k)|^2]} \\ &= \frac{N^2 \mathbf{f}_d^H \mathbf{P}(0) \mathbf{f}_d}{L^2 M_t M_r (M_t M_r + 1)} \end{aligned} \tag{8}$$

where $\mathbf{P}(k) = E[\mathbf{G}^* \mathbf{G}^T \mathbf{f}_k \mathbf{f}_{k+d}^H \mathbf{G}^* \mathbf{G}^T]$. By some mathematical manipulations, $\mathbf{P}(0)$ is denoted as the $L \times L$ circulant matrix which has $[\frac{(M_t M_r + L) M_t M_r}{N}, \frac{(M_t M_r)^2}{N}, \cdots, \frac{(M_t M_r)^2}{N}]$ as its first row.

In a similar manner to the subchannel correlation, the correlation between beamforming vectors is defined as

$$\rho_{\mathbf{w}}(d) = E[|\mathbf{w}^H(k+d)\mathbf{w}(k)|^2] \tag{9}$$

where $\|\mathbf{w}(k)\| = 1$ for all k. Since the optimal beamforming vector $\mathbf{w}(k)$ is the dominant right singular vector of $\mathbf{H}(k)$, it is not easy to express $\mathbf{w}(k)$ in terms of $\mathbf{h}(k)$ in (4). This makes it difficult to obtain a closed-form expression for $\rho_{\mathbf{w}}(d)$.

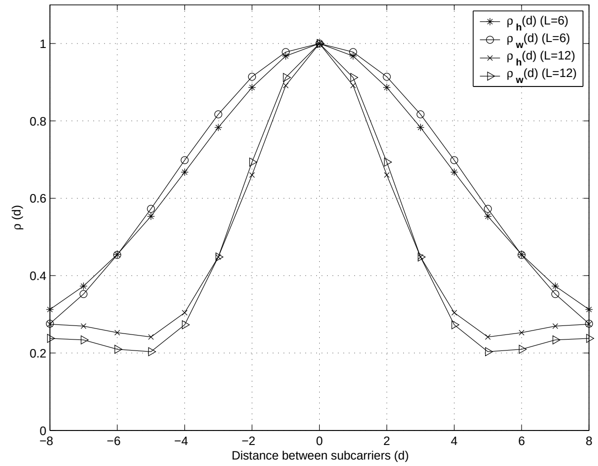

Fig. 2. Comparison between $\rho_{\mathbf{h}}(d)$ with one receive antenna and $\rho_{\mathbf{w}}(d)$ with two receive antennas when $M_t = 4$, $N = 64$, and $L = 6$ or 12.

As an alternative, the correlation between beamforming vectors are examined through computer simulations. Fig. 2 shows $\rho_{\mathbf{w}}(d)$ when $M_t = 4$, $M_r = 2$, $N = 64$, and $L = 6$ or 12. For comparison, $\rho_{\mathbf{h}}(d)$ given by (8) were plotted for $M_t = 4$, $M_r = 1$, $N = 64$, and $L = 6$ or 12. Of particular interest is that the correlation between beamforming vectors $\rho_{\mathbf{w}}(d)$ is very similar to the channel correlation $\rho_{\mathbf{h}}(d)$ with one receive antenna. In [12], it was shown that the distribution of the optimal beamforming vector is independent of the number of receive antennas. Thus, we can use the coherence bandwidth predicted by $\rho_{\mathbf{h}}(d)$ with one receive antenna to estimate the coherence of the beamforming vectors.

3.2. Proposed Interpolation Based Beamforming Method

A simple beamforming method for using the correlation of the beamforming vectors is to combine the neighboring subcarriers into a cluster and use the beamforming vector corresponding to the center subcarrier in the cluster. This method will be referred to as *clustering* [1]. If we combines K subcarriers into one cluster, the amount of feedback information is reduced to $1/K$. However, as the cluster size K increases, the beamforming performance is significantly degraded because of the distortion experienced by the subcarriers near the cluster boundary.

To mitigate the performance degradation by clustering, we consider a new beamforming method that reconstructs the beamforming vectors for all subcarriers through interpolation. In the proposed scheme, the receiver subsamples the

[1] In [20], a clustering scheme based on the channel correlation was proposed to reduce the feedback information for adaptive modulation.

subcarriers, i.e. selects a fraction of OFDM subcarriers, and evaluates the optimal beamforming vectors for the selected subcarriers. Suppose that the channel matrices $\{\mathbf{H}(k), 1 \leq k \leq N\}$ are known to the receiver and that N is divided by the subsampling rate K. The receiver evaluates the optimal beamforming vectors for the selected subcarriers $\{\mathbf{w}(1), \mathbf{w}(K+1), \mathbf{w}(2K+1), \cdots, \mathbf{w}(N-K+1)\}$ and sends them back to the transmitter. Then the transmitter evaluates the beamforming vectors for all the subcarriers by interpolating the conveyed vectors. Since the beamforming vectors have unit norm for power constraint reasons, it is natural to employ the spherical interpolation algorithms in [16]–[18]. Unfortunately, these algorithms cannot be applied to our approach for the following reason. Recall the effective channel gain $\Gamma(k) = \|\mathbf{H}(k)\mathbf{w}(k)\|^2$ in (2). When $\mathbf{w}(k)$ is the optimal beamforming vector maximizing the effective channel gain, $e^{j\phi}\mathbf{w}(k)$ also maximizes the effective channel gain. In other words, *the optimal beamforming vector is not a unique point but a line on the unit sphere*. As a consequence, algorithms that are used to compute the optimal beamforming vector, e.g. the singular value decomposition, typically choose the phase to force the first coefficient of each $\mathbf{w}(k)$ to be real. The phase, however, has a dramatic impact on the resulting interpolations. This makes it difficult to directly apply the spherical interpolation algorithms to the interpolation of beamforming vectors.

To overcome this difficulty, we propose a new algorithm through modification of the interpolator in [18]. Given $\{\mathbf{w}(1), \mathbf{w}(K+1), \mathbf{w}(2K+1), \cdots, \mathbf{w}(N-K+1)\}$, the proposed interpolator is expressed as

$$\hat{\mathbf{w}}(lK+k;\theta_l) = \frac{(1-c_k)\mathbf{w}(lK+1) + c_k\{e^{j\theta_l}\mathbf{w}((l+1)K+1)\}}{\|(1-c_k)\mathbf{w}(lK+1) + c_k\{e^{j\theta_l}\mathbf{w}((l+1)K+1)\}\|} \tag{10}$$

where $c_k = (k-1)/K$ is the linear weight value, $\mathbf{w}(N+1) = \mathbf{w}(1)$, $1 \leq k \leq K$, θ_l is a parameter for phase rotation with $0 \leq l \leq N/K - 1$. Note that $\{\mathbf{w}(k), N-K+1 \leq k \leq N\}$ are obtained by $\mathbf{w}(N-K+1)$ and $e^{j\theta_{N/K-1}}\mathbf{w}(N+1)$ (equivalently $e^{j\theta_{N/K-1}}\mathbf{w}(1)$). While the spherical interpolators in [16]–[18] only utilizes $\mathbf{w}(lK+1)$ and $\mathbf{w}((l+1)K+1)$, the proposed interpolator evaluates the beamforming vector from $\mathbf{w}(lK+1)$ and $e^{j\theta_l}\mathbf{w}((l+1)K+1)$. Essentially the role of θ_l is to remove the distortion caused by the arbitrary phase rotation of the optimal beamforming vectors.

3.3. Phase Optimization for the Proposed Interpolator

In this subsection, the proposed interpolator is optimized by determining the phases $\{\theta_l, 0 \leq l \leq N/K - 1\}$ in the sense of maximizing the capacity. The determined phase parameters are conveyed back along with the selected beamforming vectors to the transmitter.

In (10), θ_l is only used for computing $\{\hat{\mathbf{w}}(lK+k), 1 \leq k \leq K\}$, and thus the optimal θ_l maximizing the capacity can be found by

$$\theta_l = \arg\max_{\theta} \sum_{k=1}^{K} \log_2 \left\{1 + \frac{\|\mathbf{H}(lK+k)\hat{\mathbf{w}}(lK+k;\theta)\|^2}{N_0}\right\} \tag{11}$$

where $0 \leq l \leq N/K-1$. Due to the normalization factor in $\hat{\mathbf{w}}(lK+k;\theta)$, it is not easy to get a closed-form solution for (11). Instead, we use a numerical grid search by modifying (11) as

$$\theta_l = \arg\max_{\theta\in\Theta} \sum_{k=1}^{K} \log_2 \left\{1 + \frac{\|\mathbf{H}(lK+k)\hat{\mathbf{w}}(lK+k;\theta)\|^2}{N_0}\right\}. \tag{12}$$

where $\Theta = \{0, \frac{2\pi}{P}, \frac{4\pi}{P}, \cdots, \frac{2(P-1)\pi}{P}\}$, and P is the number of quantized levels which determines the performance and complexity of the search.

As an alternative to the grid search, (11) is simplified by considering the average channel capacity. In general, $\hat{\mathbf{w}}(lK+K/2+1)$ experiences the largest distortion by interpolation, because the subcarrier $(lK+K/2+1)$ is the furthest from the subcarriers $(lK+1)$ and $((l+1)K+1)$ with feedback of the beamforming vectors. Using this observation, (11) can be approximated to maximize the capacity of the subcarrier $(lK+K/2+1)$. Then, θ_l is approximately obtained by

$$\theta_l = \arg\max_{\theta} \log_2 \left\{1 + \frac{\|\mathbf{H}(lK+K/2+1)\hat{\mathbf{w}}(lK+K/2+1;\theta)\|^2}{N_0}\right\}. \tag{13}$$

By the concavity of log function, this equation is equivalently rewritten as

$$\begin{aligned}\theta_l &= \arg\max_{\theta} \|\mathbf{H}(lK+K/2+1)\hat{\mathbf{w}}(lK+K/2+1;\theta)\|^2 \\ &= \arg\max_{\theta} \frac{(\mathbf{w}_1 + e^{j\theta}\mathbf{w}_2)^H\mathbf{R}(\mathbf{w}_1 + e^{j\theta}\mathbf{w}_2)}{\|\mathbf{w}_1 + e^{j\theta}\mathbf{w}_2\|^2}\end{aligned} \tag{14}$$

where $\mathbf{w}_1 = \mathbf{w}(lK+1)$, $\mathbf{w}_2 = \mathbf{w}((l+1)K+1)$, and $\mathbf{R} = \mathbf{H}^H(lK+K/2+1)\mathbf{H}(lK+K/2+1)$. By differentiating the cost function in (14) with respect to θ, we get

$$\mathrm{Imag}(\alpha_1 e^{j\theta} + \alpha_2) = 0 \tag{15}$$

where $\mathrm{Imag}(\cdot)$ means the imaginary part of $(\cdot)$, $\alpha_1 = (\mathbf{w}_1^H\mathbf{R}\mathbf{w}_2)(\mathbf{w}_1^H\mathbf{w}_1 + \mathbf{w}_2^H\mathbf{w}_2) - (\mathbf{w}_1^H\mathbf{R}\mathbf{w}_1)(\mathbf{w}_1^H\mathbf{w}_2) - (\mathbf{w}_2^H\mathbf{R}\mathbf{w}_2)(\mathbf{w}_1^H\mathbf{w}_2)$, and $\alpha_2 = 2(\mathbf{w}_1^H\mathbf{R}\mathbf{w}_2)(\mathbf{w}_2^H\mathbf{w}_1)$. It can be shown that $|\alpha_1| \geq |\mathrm{Imag}(\alpha_2)|$ so that there always exists θ satisfying (15). Suppose that $\sin^{-1}(x) \in [-0.5\pi, 0.5\pi)$ and let denote $\alpha_i = |\alpha_i|e^{j\phi_i}$. Then the

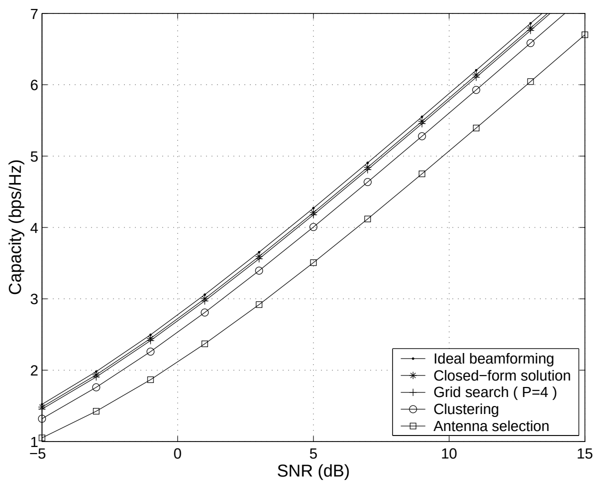

Fig. 3. Capacity without quantization of beamforming vectors when $M_t = 4$, $M_r = 2$, $N = 64$, and $K = 8$. Grid search in (12) and closed-form solution in (14) were used.

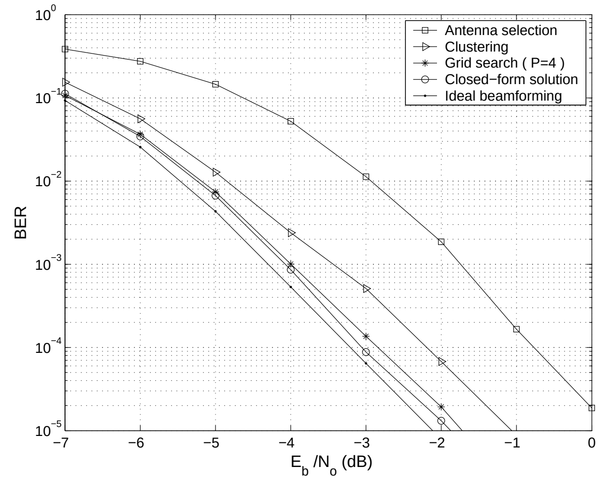

Fig. 4. Comparison of BER performance when $M_t = 4$, $M_r = 2$, $N = 64$, $K = 8$, and channel coding was used.

solutions of (15) are given by

$$\vartheta_1 = -\phi_1 + \epsilon \tag{16}$$

$$\vartheta_2 = \begin{cases} -\phi_1 + \pi - \epsilon & \text{if } \epsilon \geq 0 \\ -\phi_1 - \pi - \epsilon & \text{if } \epsilon < 0 \end{cases} \tag{17}$$

where $\epsilon = \sin^{-1}\left(-\frac{|\alpha_2|}{|\alpha_1|}\sin\phi_2\right)$. Also, the derivative of (15) is denoted as

$$\frac{d}{d\theta} j\{\alpha_1 e^{j\theta} + (\alpha_2 - \alpha_2^*) - \alpha_1^* e^{-j\theta}\} = -2\,\mathrm{Real}(\alpha_1 e^{j\theta}) \tag{18}$$

where $\mathrm{Real}(\cdot)$ means the real part of $(\cdot)$. Since (18) gives the sign of the second derivative of the cost function, the optimal θ_l maximizing the cost function is one of the solutions ϑ_1 and ϑ_2 such that $-2\ \mathrm{Real}(\alpha_1 e^{j\theta_l})$ is negative. This solution will be referred to as the *closed-form solution*.

4. Simulation Results

Through computer simulations, we compare the proposed beamforming method with antenna selection diversity [7], the simple clustering method, and the ideal beamforming with feedback of all beamforming vectors. In the simulation, we consider a MIMO-OFDM system with parameters: $M_t = 4$, $M_r = 2$, $N = 64$, $K = 8$. We assumed that each discrete-time channel impulse response had 6 taps ($L = 6$) with a uniform power delay profile and i.i.d. complex Gaussian distribution with zero mean and variance $1/L$; the channels between different transmit and receiver antenna pairs were independent; $\mathbf{n}(k)$ was i.i.d. complex Gaussian with zero mean; the feedback channel had no delay and no transmission error; receiver used MRC with perfect channel knowledge; quadrature phase shift keying (QPSK) modulation was used for BER simulations; the transmit power was identically assigned to all the OFDM subcarriers. Every point of the simulation results was obtained by averaging over more than 5000 independent realizations of channel and noise.

Fig. 3 presents the channel capacity obtained by various beamforming techniques. In the figure, "grid search" and "closed-form solution" describe the proposed interpolators using the grid search in (12) and the closed-form solution in (14), respectively. The proposed grid search and closed-form solution had higher capacity than the clustering and antenna selection, but showed slight loss compared to the ideal beamforming. The proposed methods performed within 0.3 dB of the ideal beamforming, while the clustering had about 1 dB loss at capacity=4 bps/Hz.

To further evaluate the performance of the proposed scheme, the BER values with channel coding and interleaving are presented in Fig. 4. For channel coding, we used a convolutional code with generator polynomials $g_0 = 133_8$ and $g_1 = 171_8$ with coding rate $1/2$, along with the interleaver and deinterleaver defined in [15], and soft Viterbi decoding. The frame length was 30 OFDM symbols and each OFDM symbol transmitted 64 bits. The channel was assumed to be fixed for a frame and randomly varied between frames. As expected in the capacity result, the proposed closed-form solution performed comparable to the grid search with $P = 4$, and exhibited 0.8 dB gain over the clustering and 2.2 dB gain over selection diversity at BER=10^{-4}. As compared with ideal beamforming, the E_b/N_0 loss of the proposed was within 0.2 dB.

5. Conclusions

A new transmit beamforming scheme for MIMO-OFDM was proposed. Using the correlation between neighboring beamforming vectors, the proposed scheme reduces the

feedback requirements for a closed-loop MIMO-OFDM system. A new spherical interpolator was proposed to interpolate beamforming vectors and the proposed interpolator can be optimized by adjusting the parameters for phase rotation in terms of an effective channel gain or capacity. Through computer simulations, it was shown that the proposed method has considerable benefits in performance and the amount of feedback information compared to existing diversity techniques. Our work proposed herein is limited to a MIMO system that transmits a single data stream per subcarrier. Extending this approach to the case where multiple beamforming vectors, combining with spatial multiplexing as for example in [22], remains a future research topic. Accounting for imprecision of channel state information, as in [10], is also of interest.

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Spherical Linear Interpolation for Transmit Beamforming in MIMO-OFDM Systems with Limited Feedback

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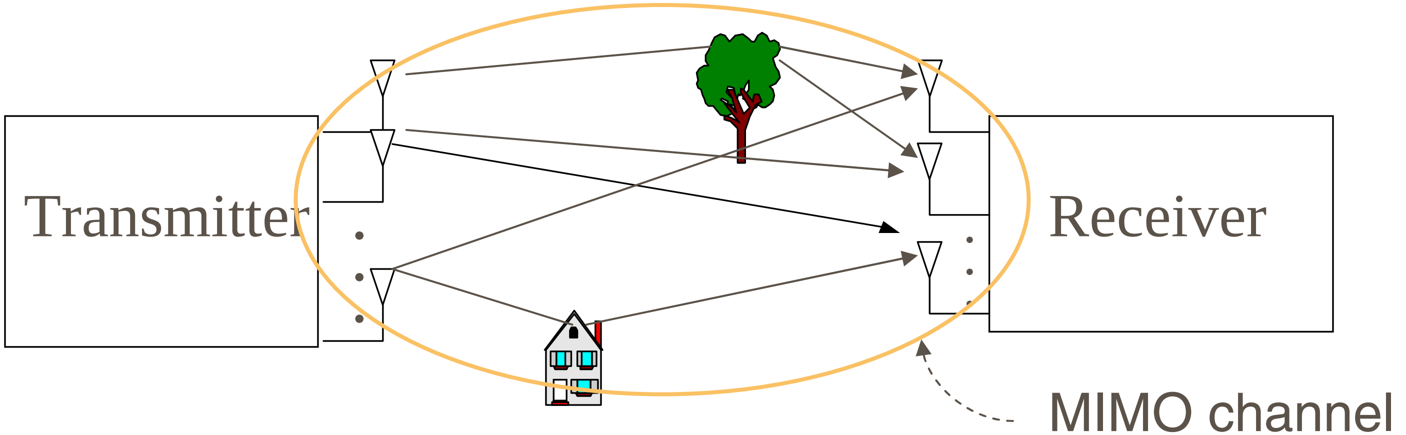
Outline

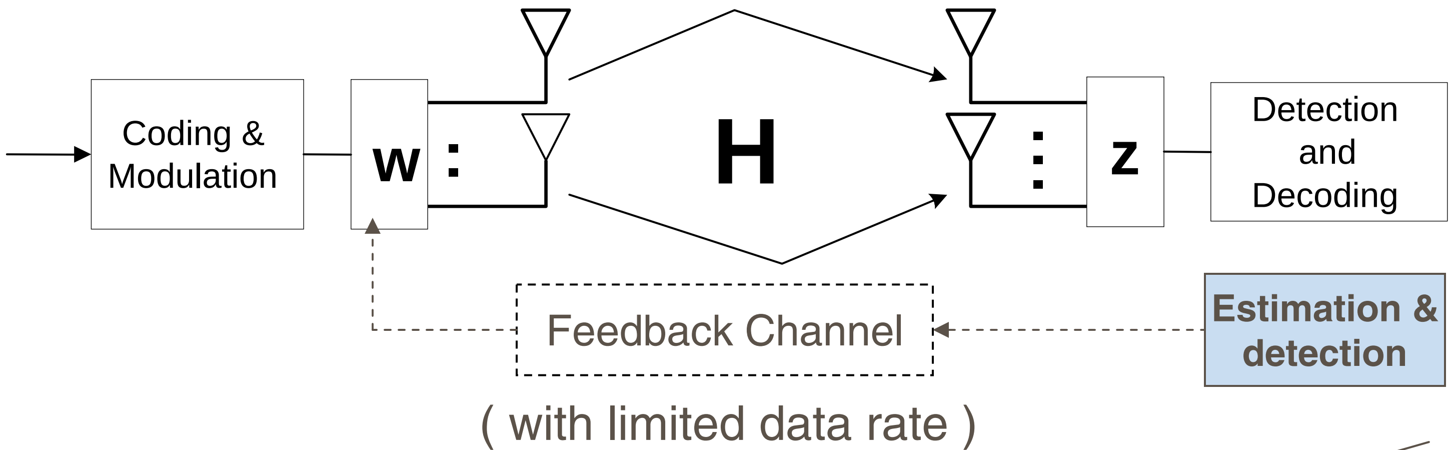
- Overview
  - Diversity techniques in MIMO systems
  - Closed-loop transmit beamforming for MIMO-OFDM
- Problem Statement
- Proposed Beamforming Scheme
  - Proposed spherical interpolator
  - Optimization of phase rotation
  - Beamformer quantization + interpolation
- Numerical Simulations and Discussions

Transmit Diversity in MIMO Systems



- No channel state information (CSI) at Transmitter
  - Space-time codes [Tarokh et al. 1998]
- Full CSI at Transmitter
  - Antenna selection [Wittneben et al.1994], [Heath et al. 1998]
  - Maximum ratio transmission [Lo 1999], [Andersen 2000]
- Partial CSI
  - Adaptive beamforming [Xia et. al. 2004]

Beamforming and Combining



Complex number

$$y = \mathbf{z}^*\mathbf{H}\mathbf{w}s + \mathbf{z}^*\mathbf{n}$$

- Covert MIMO to SISO
  - Transmit beamforming vector **w**
  - Receive combining vector **z**

Max SNR Solution (MRT/MRC)

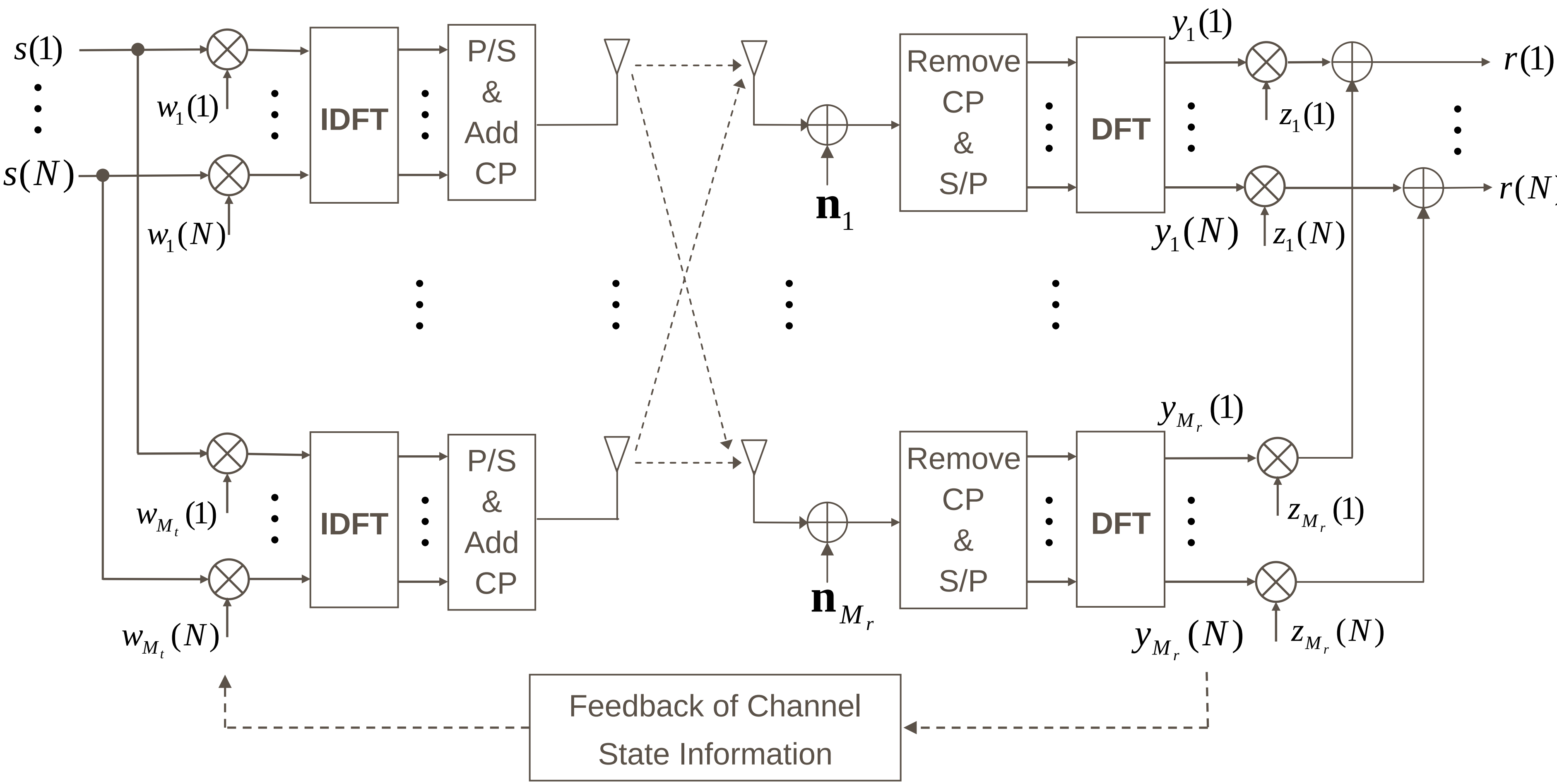
- SNR is proportional to $\frac{\|\mathbf{z}^*\mathbf{H}\mathbf{w}\|^2}{\|\mathbf{z}\|}$
- Max SNR solution is

$$\mathbf{w} = \max_{\|\mathbf{w}\|=1} \|\mathbf{H}\mathbf{w}\|$$

$$\mathbf{z} = \mathbf{H}\mathbf{w}$$

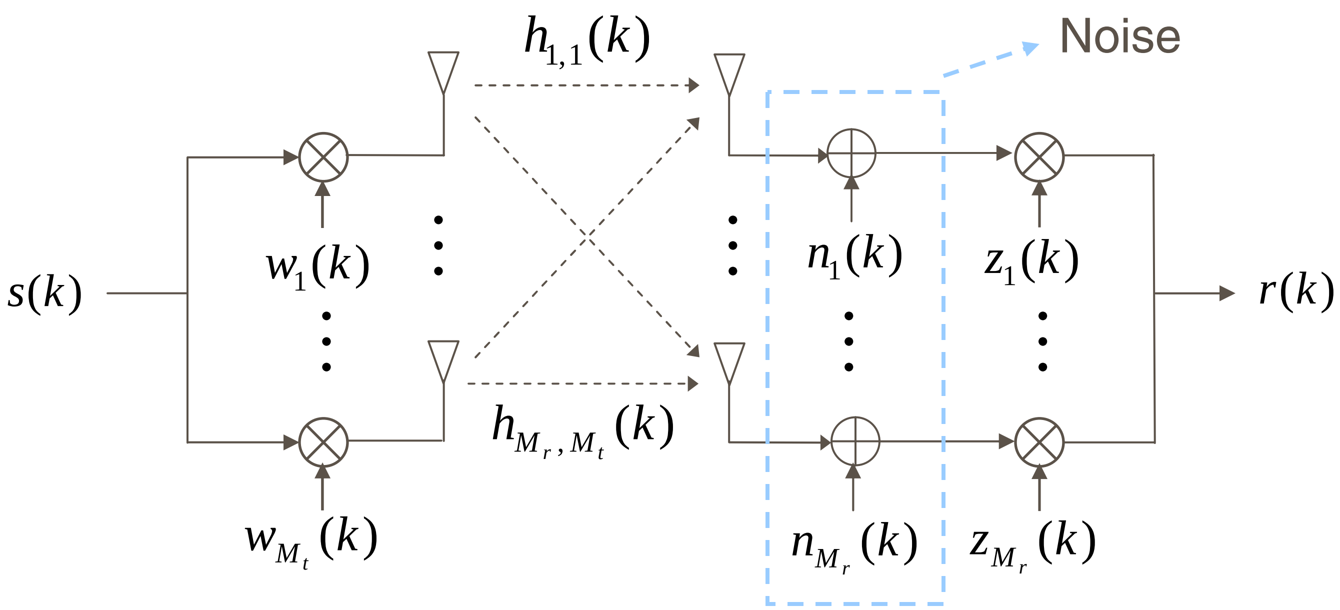
- Solution is not unique: if **w** is optimal, so is **w**$e^{j\Theta}$
- Other solutions possible (constrain **w**)
  - Selection diversity
  - Equal gain combining / transmission

Beamforming in MIMO-OFDM



Signal Model for One Subcarrier

- Equivalent model for subcarrier *k*
  (identical with a narrowband MIMO case)



$$r(k) = \mathbf{z}^H(k)\{\mathbf{H}(k)\mathbf{w}(k)s(k) + \mathbf{n}(k)\}$$

Problem Summary

- Beamforming in MIMO-OFDM requires $\{\mathbf{w}(n)\}_{n=0}^{N-1}$
  - Feedback requirements $\propto$ Number of subcarriers
- How can we reduce the number of vectors fed back?
  - Exploit correlation between vectors
  - Send back fraction of vectors
  - Use “smart” interpolation
- How can limit the feedback for each vector?
  - Use quantized beamforming [Love et. al. 2003]

Correlation Between Subcarriers

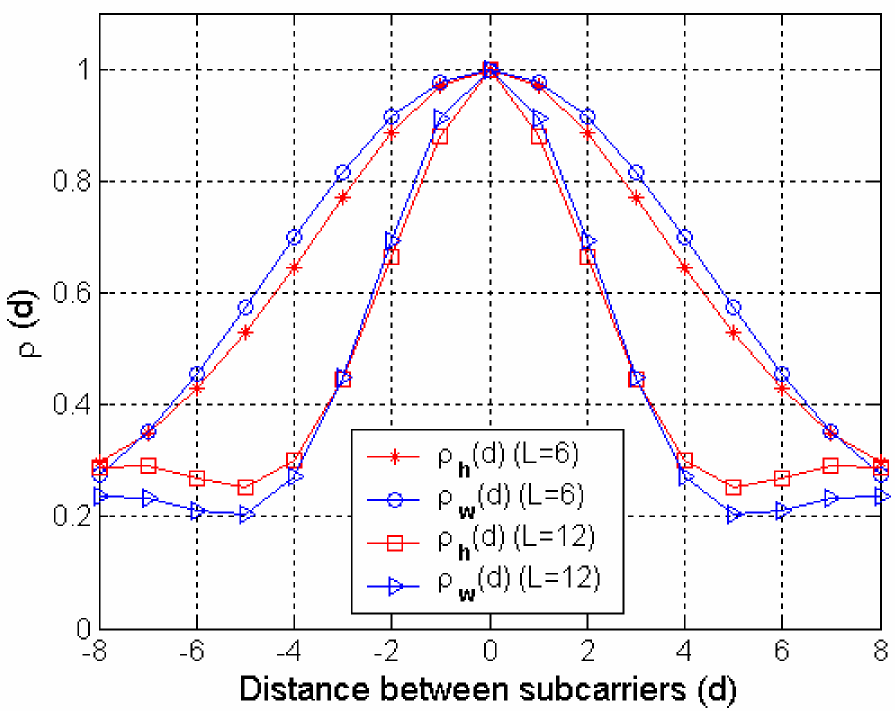
- Subchannel correlation

$$\rho_{\mathbf{h}}(d) = \frac{E[|\mathbf{h}^H(k+d)\mathbf{h}(k)|^2]}{E[\|\mathbf{h}(k)\|^4]}$$

where $\mathbf{h}(k) = \text{vec}(\mathbf{H}(k))$

- Beamformer correlation

$$\rho_{\mathbf{w}}(d) = E[|\mathbf{w}^H(k+d)\mathbf{w}(k)|^2]$$



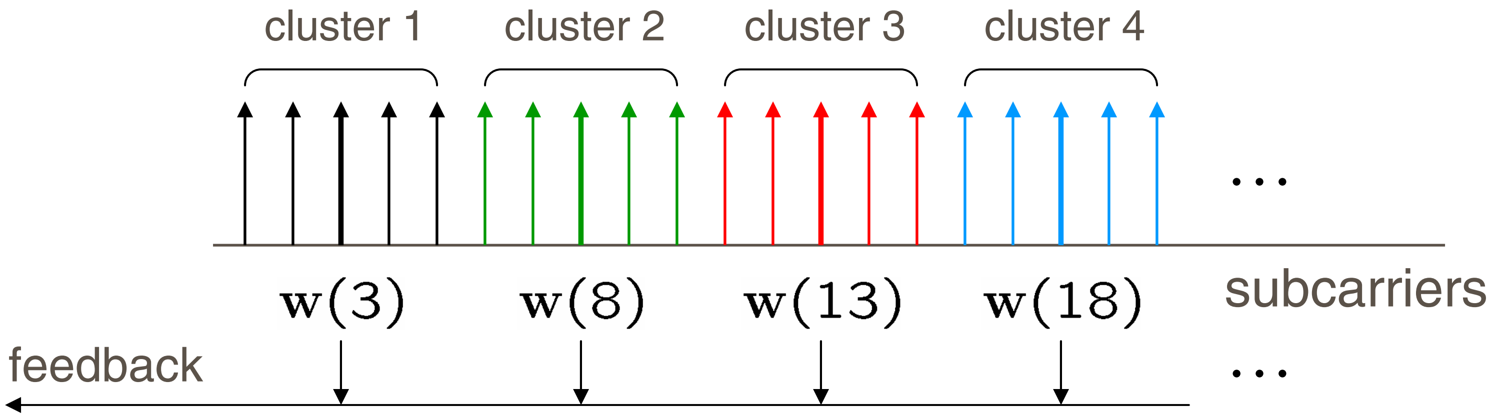
- Simulations
  - $\rho_{\mathbf{h}}(d)$: M_t=4, M_r=1, N=64, K=8, L={6,12}
  - $\rho_{\mathbf{w}}(d)$: M_t=4, M_r=2, N=64, K=8, L={6,12}

subsampling rate

number of channel taps

Clustering of Subcarriers

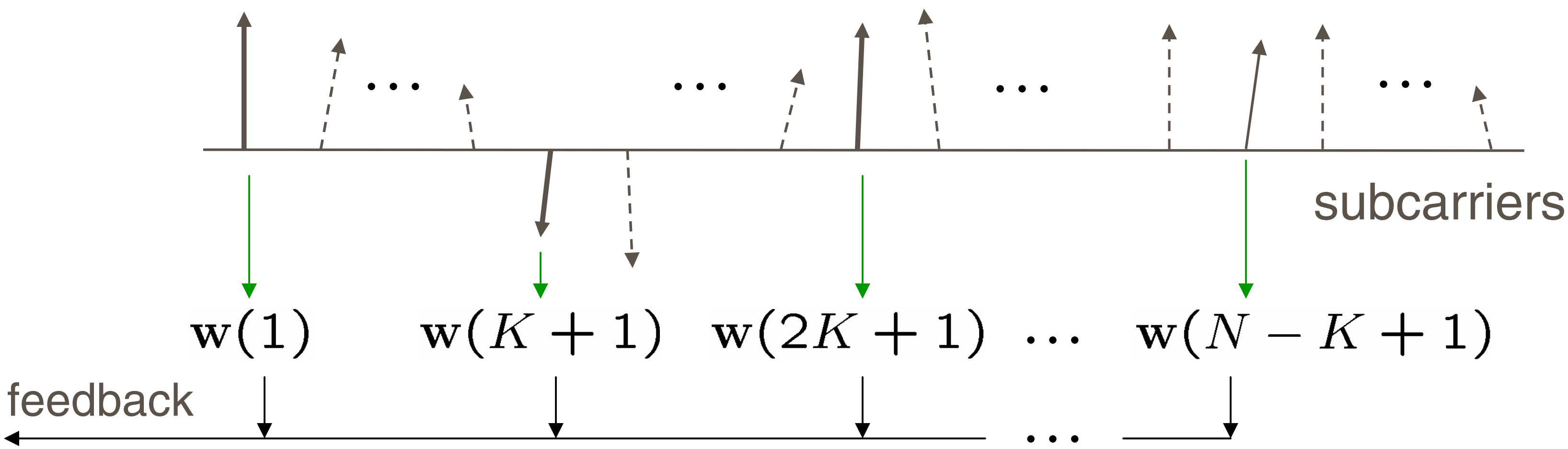
- Clustering (*K*=5)



- Disadvantages
  - Performance degradation in cluster boundary
  - Cluster size (or feedback reduction) is limited

Proposed Beamforming Method

- Subsampling of beamforming vectors



- Reconstruction of beamforming vectors
  - Transmit power constraints force $\|\mathbf{w}(k)\| = 1$

  $\longrightarrow$ Spherical interpolation

Conventional Spherical Interpolation

- Spherical averaging [Watson 1983], [Buss et al. 2001]

$$\hat{\mathbf{v}} = \frac{\sum_{i=0}^{p} b_i \mathbf{v}_i}{\|\sum_{i=0}^{p} b_i \mathbf{v}_i\|}$$

  - p is the order, $\|\mathbf{v}\| = 1$, $\sum_{i=0}^{p} b_i = 1$, and $b_i \geq 0$

- Nonuniqueness of optimal beamforming vectors

$$\mathbf{w}(k) \text{ is optimal} \quad \longleftrightarrow \quad e^{j\theta}\mathbf{w}(k) \text{ is optimal}$$

  - Random phase rotation θ has significant impact on interpolations
  - Performance degradation of spherical interpolators

Proposed Interpolator

- Spherical Linear Interpolator with phase rotation

$$\hat{\mathbf{w}}(k,\theta_1) = \frac{(1-c_k)\mathbf{w}(1) + c_k\{\boxed{e^{j\theta_1}}\mathbf{w}(K+1)\}}{\|(1-c_k)\mathbf{w}(1) + c_k\{\boxed{e^{j\theta_1}}\mathbf{w}(K+1)\}\|}, \quad 1 \le k \le K$$

  - $c_i = \frac{k-1}{K}$ and θ_1 is a parameter for phase rotation.

- Optimization of phase rotation parameters
  - Maximize the diversity gain
  - Maximize the mutual information

Optimization of Phase Rotation

- Maximizing the minimum channel gain (diversity)

$$\theta_1 = \arg \max_{\theta \in [0, 2\pi)} \min\{\|\mathbf{H}(k)\hat{\mathbf{w}}(k,\theta)\|^2,\ 1 \leq k \leq K\}$$

  - Difficult to get a closed-form solution

- Grid search

$$\theta_1 = \arg \max_{\theta \in \Theta} \min\{\|\mathbf{H}(k)\hat{\mathbf{w}}(k,\theta)\|^2,\ 1 \leq k \leq K\}$$

  - $\Theta = \{0, \frac{2\pi}{P}, \frac{4\pi}{P}, \cdots, \frac{2(P-1)\pi}{P}\}$ and *P* is the number of quantized levels

Alternative Solution

- Observation
  - Subcarrier (*K*/2+1) suffers from the largest distortion
  - Subcarrier (*K*/2+1) has the worst average channel gain

- Approximation of cost function

$$
\begin{aligned}
\theta_1 &= \arg \max_{\theta \in [0,2\pi)} \|\mathbf{H}(K/2+1)\hat{\mathbf{w}}(K/2+1,\theta)\|^2 \\
&= \arg \max_{\theta \in [0,2\pi)} \frac{\|\mathbf{H}(K/2+1)\{\mathbf{w}(1)+e^{j\theta}\mathbf{w}(K+1)\}\|}{\mathbf{w}(1)+e^{j\theta}\mathbf{w}(K+1)}
\end{aligned}
$$

Closed-Form Solution

- Differentiation with respect to θ

$$\text{Imag}(\alpha_1 e^{j\theta} + \alpha_2) = 0$$

  - α_1 and α_2 are complex constants satisfying $|\alpha_1| \geq |\text{Imag}(\alpha_2)|$
  - Always, there exist two solutions.

- Second derivative

$$\frac{d}{d\theta}\text{Imag}(\alpha_1 e^{j\theta} + \alpha_2) = -2\text{Real}(\alpha_1 e^{j\theta})$$

  - The solution making the second derivative be negative is the optimal solution. $\longrightarrow$ *closed-form solution*

Optimization in Terms of Capacity

- Maximizing the capacity

$$\theta_1 = \arg \max_{\theta \in [0, 2\pi)} \sum_{k=1}^{K} \log_2 \left(1 + \frac{\|\mathbf{H}(k)\hat{\mathbf{w}}(k,\theta)\|^2}{N_0} \right)$$

- Grid search

$$\theta_1 = \arg \max_{\theta \in \Theta} \sum_{k=1}^{K} \log_2 \left(1 + \frac{\|\mathbf{H}(k)\hat{\mathbf{w}}(k,\theta)\|^2}{N_0} \right)$$

  - $\Theta = \{0, \frac{2\pi}{P}, \frac{4\pi}{P}, \cdots, \frac{2(P-1)\pi}{P}\}$

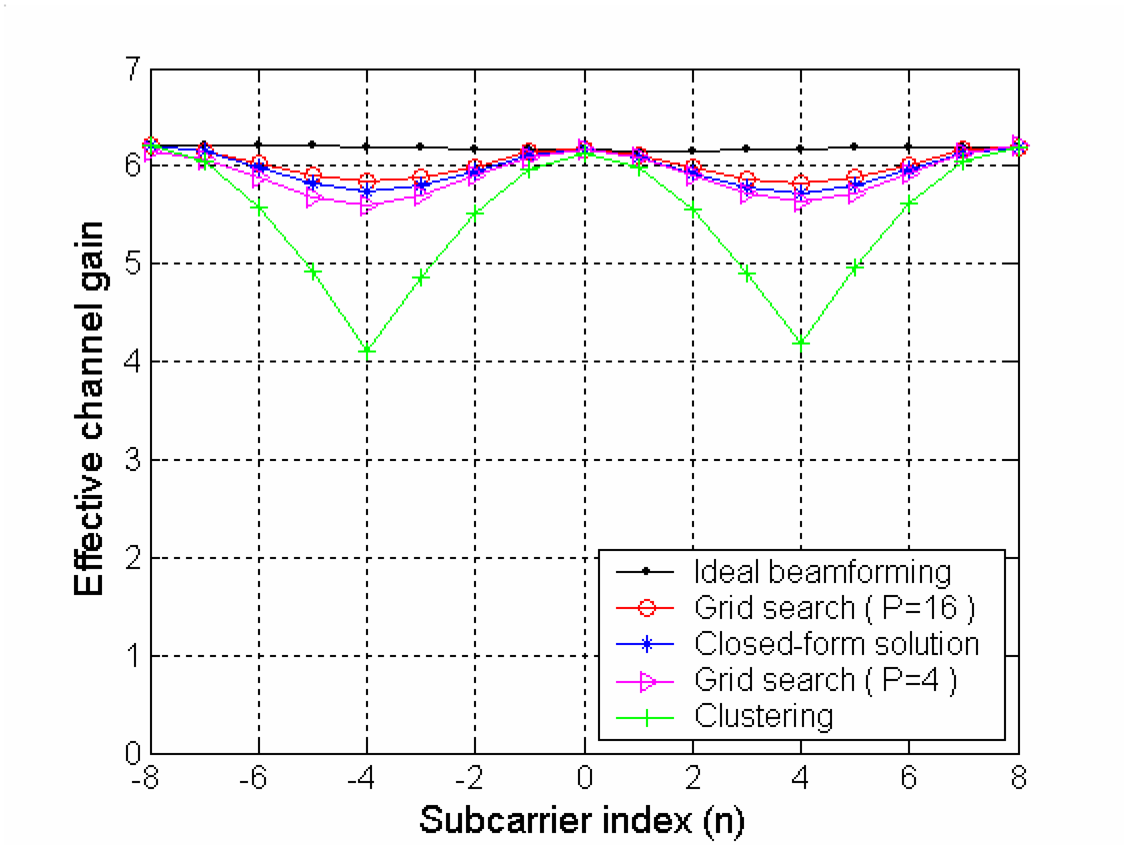
- Closed-form solution is the same as in diversity case

Simulation Environments

- MIMO channels
  - Each channel impulse response has 6 taps (L=6) with uniform profile and follows i.i.d. $CN(0,1/L)$.
  - The channels between different transmit and receiver antennas pairs are independent.
- Noise: i.i.d. with CN$(0,N_0)$
- Simulation parameters
  - M_t=4 (# of Tx antennas), M_r=2 (# of Rx antennas)
  - N=64 (# of subcarriers), K=8 (subsampling rate), L=6
- Basic assumptions
  - MRC at the receiver
  - No delay and no transmission error in the feedback channel
  - QPSK, no water-filling

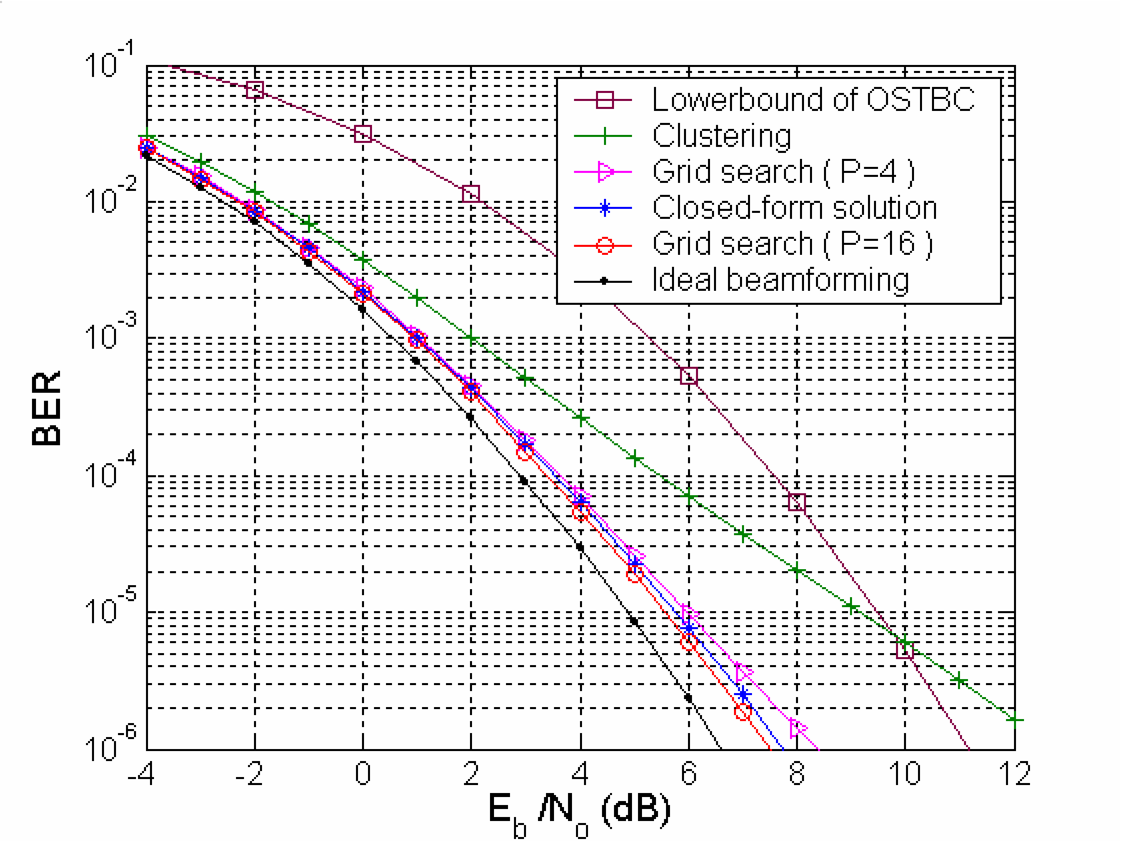
Simulation Results – Channel Gain

- OFDM subcarrier vs. effective channel gain



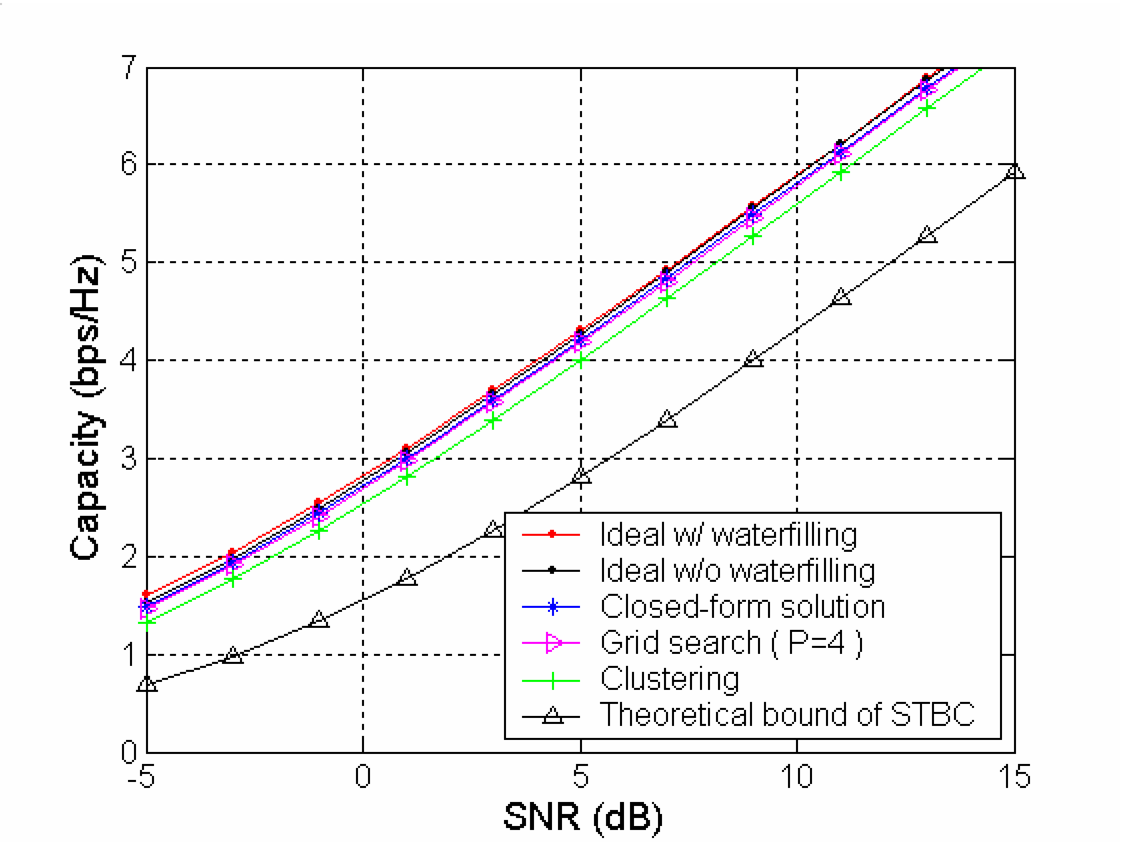
Simulation Results – Uncoded BER

- No channel coding



Simulation Results – Capacity

- No power control for diversity techniques

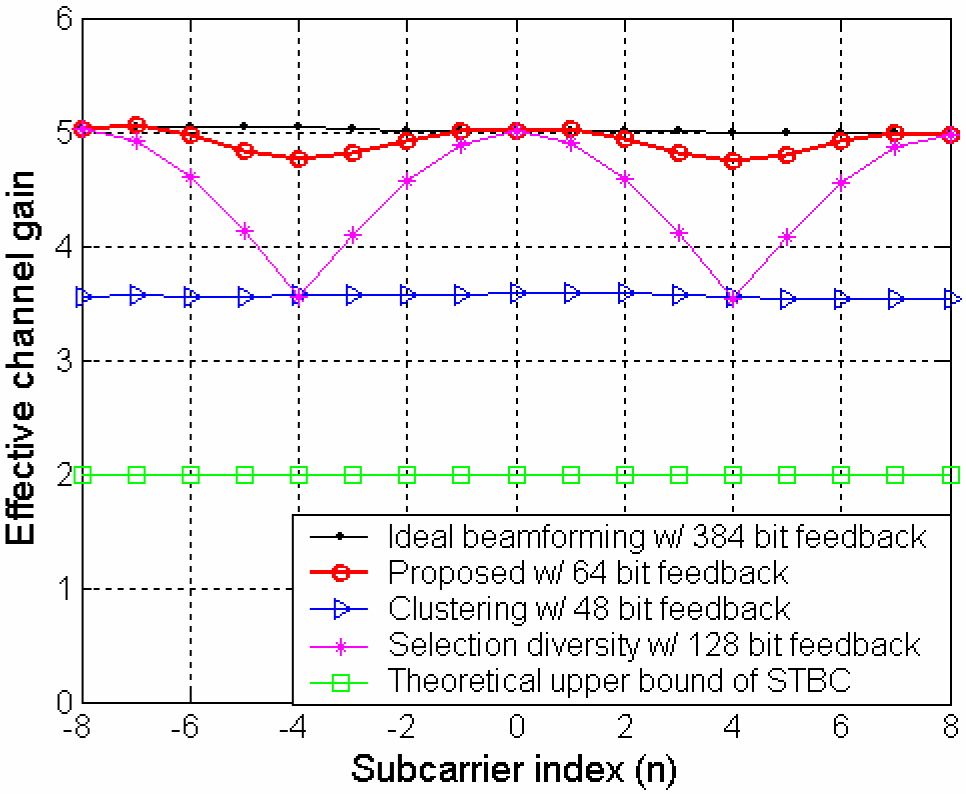


Simulation with Quantization

- Quantization of beamforming vectors
  - By using the codebook in [Love & Heath 2003]
  - 6 bits of feedback for each beamforming vector
- Phase optimization
  - Grid search with P=4 (2 bits of feedback per phase)
- Feedback requirements
  - Ideal: 6N = 384 bits / OFDM symbol
  - Clustering: 6N/K = 48 bits / OFDM symbol
  - Selection: 2N = 128 bits / OFDM symbol
  - Proposed: 8N/K = 64 bits / OFDM symbol
  - Orthogonal STBC (OSTBC): none

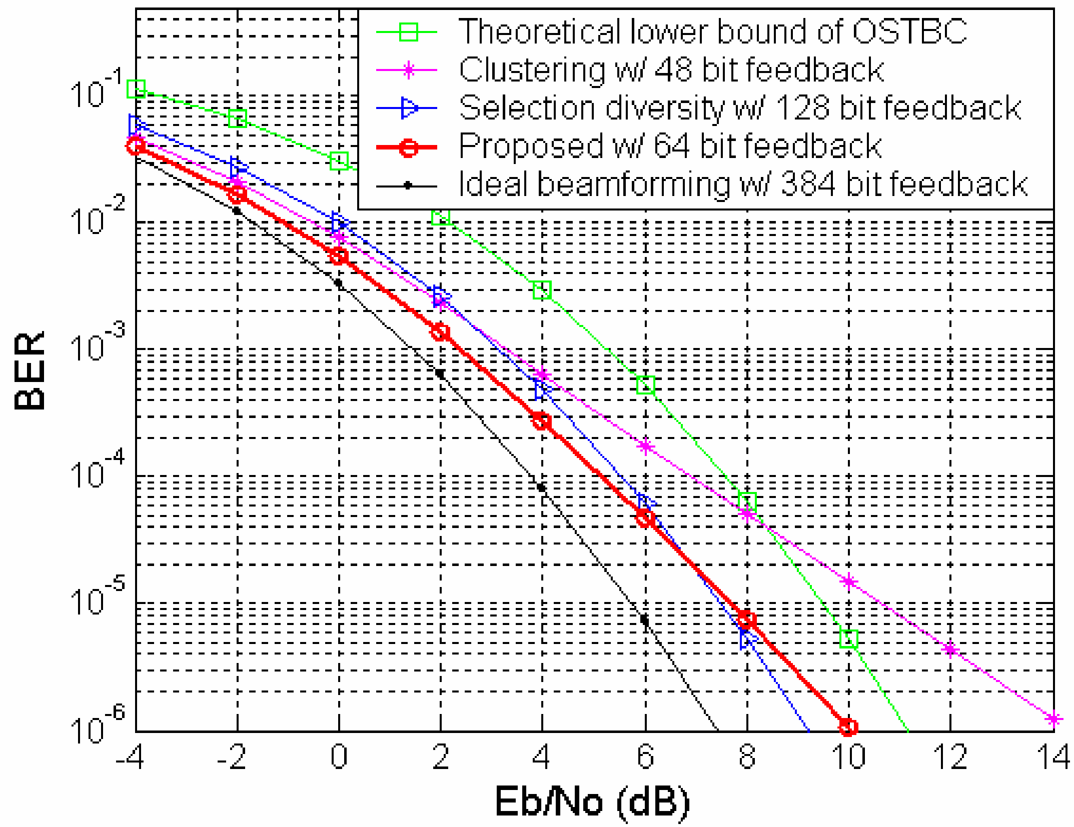
Simulation Results - Channel Gain

- M_t=4, M_r=2, N=64, K=8, L=6



Simulation Results - BER

- Selection
  - Full diversity order
  - Array gain loss
- Proposed
  - Diversity order loss
  - Additional array gain



Simulation Results - BER

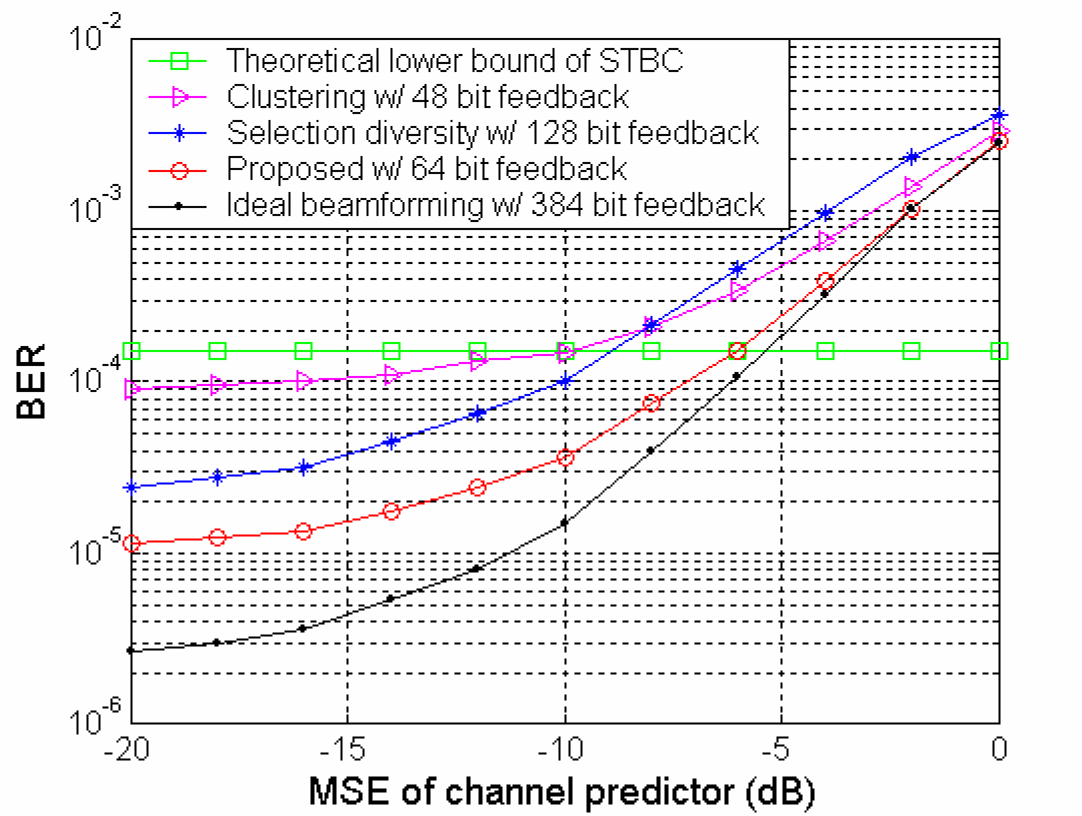
- BER degradation by channel prediction error

◆ Prediction error model

$$\tilde{\mathbf{H}}(k) = \gamma \mathbf{H}(k) + \sqrt{1-\gamma^2}\mathbf{U}(k)$$

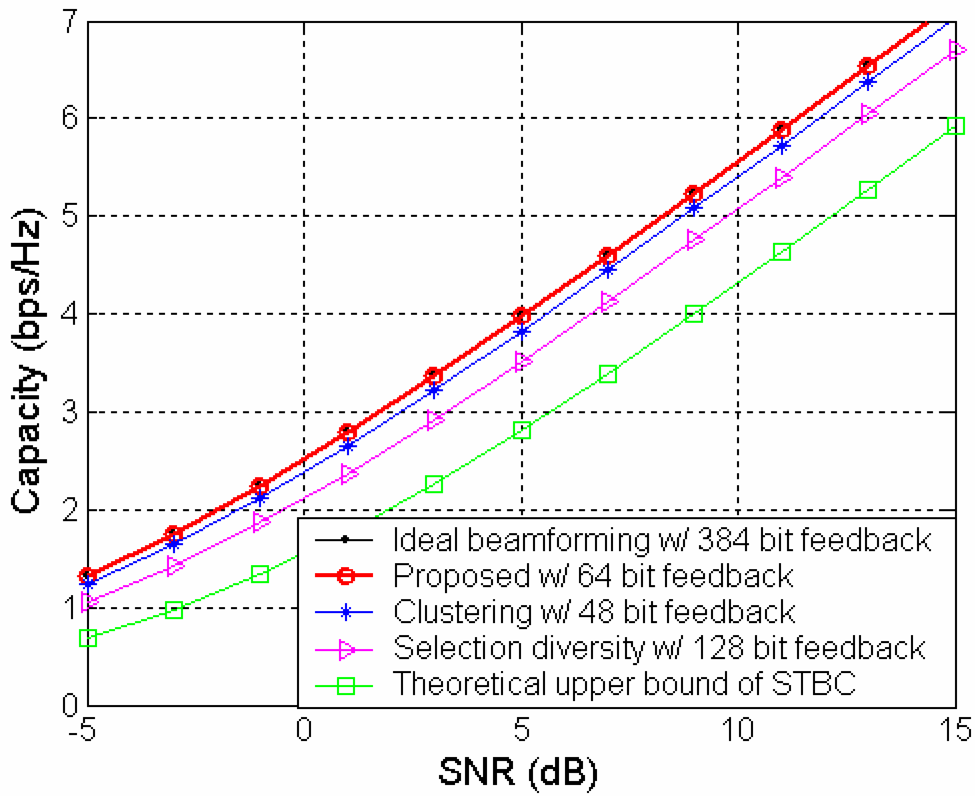
where

$\mathbf{U}(k)$ is a prediction error



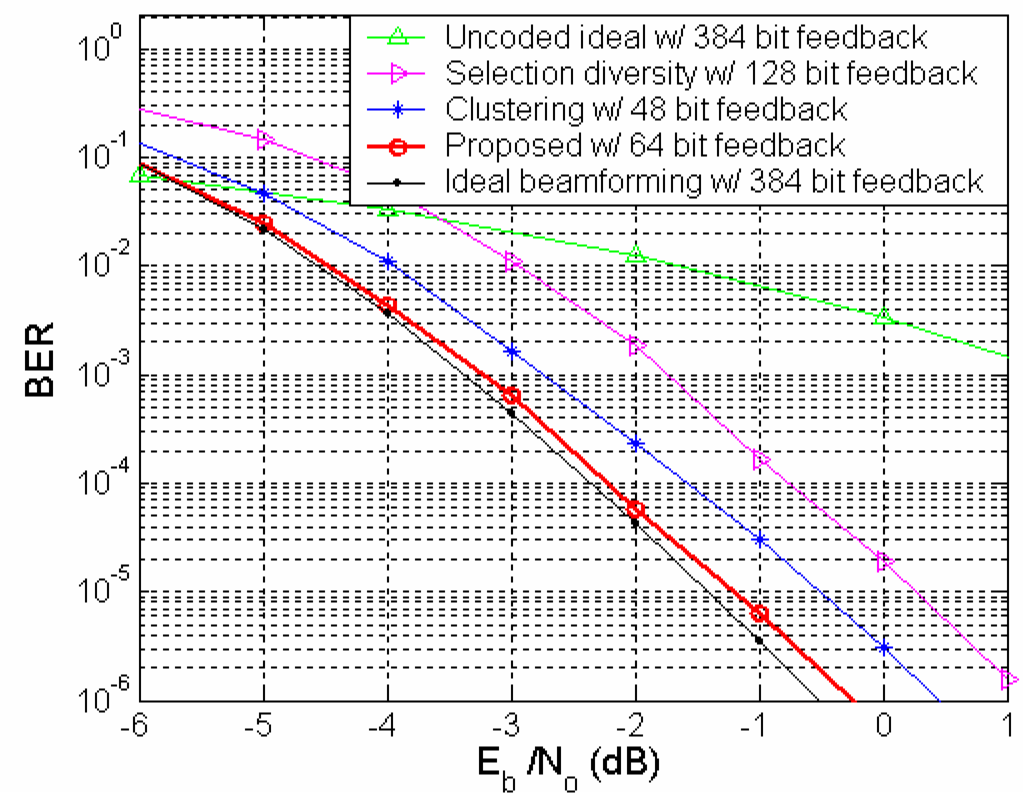
Simulation Results - Capacity

- No power control



Simulation Results – Coded BER

- 1/2 convolutional code
- Interleaving/deinterleaving
- Frame length
  - 30 OFDM symbols
- 64 bits / OFDM symbol



Conclusions

- Summary
  - Interpolation for transmit beamforming in MIMO-OFDM proposed
  - Relates to problem of spherical quantization
- Future work
  - Proposed system does not obtain MIMO capacity
  - Develop “interpolated” waterfilling solutions